Diameters of vortex spirals in three-dimensional turbulence

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It has been proposed by a number of authors that fully developed turbulence contains spiral-like vorticity distributions that wrap up around strained vortex tubes, but numerical simulations seem to show otherwise. In the present paper, we suggest an explanation for the numerical results. Using stability considerations and numerical results for the Reynolds numbers of the most intense vortex tubes, we obtain an estimate for the characteristic diameter of the largest spirals as a function of the Reynolds number. We find that this diameter decreases rapidly, relative to the integral scale, as the Reynolds number tends to infinity.

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It is well known that the Kelvin-Helmholtz instability generates spiral-like structures in vortex sheets, and this has motivated a number of authors [1–6] to consider spiral models for the inertial-range structure of turbulence. The interest in such models is to a large extent due to the striking result that a model with sheets wrapping around axially strained vortex tubes leads to a Kolmogorov $-\frac{5}{3}$ power law for the inertial-range energy spectrum [1].

However, direct numerical simulations [7–9] show that the vorticity field of homogeneous turbulence at Taylor microscale Reynolds numbers $R_\lambda \approx 150-170$ contains no significant spiral structure at all. Since laminar flows and flows in transition to turbulence [10] do contain spiral-like structures, the characteristic spiral diameter apparently decreases rapidly as R_λ increases.

[The authors of Ref. [10] did not calculate R_λ [14], which makes it difficult to compare with Refs. [7–9]. However, the energy spectrum shown in Ref. [10] falls off like a power law over a range of wave numbers that is consistent with a value of R_λ well below 50.]

In the present paper, we consider the effect of small-scale disturbances on spiral diameters. In previous works, it has been assumed (without discussion apparently) that such disturbances have no effect on the diameters. We find, on the contrary, that disturbances have a strong effect on the Reynolds-number scaling of spiral diameters.

Specifically, we argue in the following that

$$D \propto R_\lambda^{-5/4} L, \tag{1}$$

where D is the characteristic spiral diameter and L is the integral scale. To derive this result, we assume that D scales like the largest distance from vortex tubes at which these deform the fluid faster than average small-scale disturbations do. The Reynolds-number scaling of this distance can in turn be estimated by means of a numerical result for the Reynolds-number scaling of the circulation-based Reynolds number of the most intense vortex tubes [9]. However, the value of the exponent in Eq. (1) is only suggestive; the main point is that D/L decreases rapidly as R_λ tends to infinity.

Let us first estimate the largest rate at which a given tube can deform the fluid at distances much larger than the Kolmogorov disspation scale η. Reference [9] reports that the most intense tubes have diameters of order η, so we may approximate the flow at distances much larger than η by that around a vortex line with similar total circulation Γ; such an approximation suffices when seeking an order of magnitude estimate of the rate at which the tube deforms the fluid. Let now $u(r)$ denote the numerical value of the velocity at a distance r from the vortex line,

$$u(r) = \frac{\Gamma}{2\pi r}. \tag{2}$$

Such a vortex line deforms the fluid at distance r at the rate

$$\sigma(r) = \frac{|\Gamma|}{2\pi r^2}. \tag{3}$$

Introducing a circulation-based Reynolds number R_ν,

$$R_\nu = \frac{|\Gamma|}{\nu}, \tag{4}$$

we get

$$\sigma(r) = \frac{R_\nu \nu}{2\pi r^2}. \tag{5}$$

We next estimate the rate at which small-scale disturbances deform the fluid. In homogeneous isotropic turbulence, the smallest velocity disturbances typically deform the fluid at rates comparable with the root-mean-square vorticity (recall that the fastest growing perturbations of vortex sheets are those at small scales [12]). Let us assume that small-scale disturbances deform vortex sheets at similar rates. Thus, D scales like the distance at which the deformation rates in-

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duced by tubes are of the same magnitude as the root-mean-square vorticity of the turbulence,

$$\sigma(D) \propto \langle \omega^2 \rangle^{1/2}, \tag{6}$$

where $\propto$ denotes proportionality as $\langle \omega^2 \rangle$ increases to infinity. Using Eq. (5), we get

$$D \propto \langle \omega^2 \rangle^{-1/4} \sqrt{R_v \nu}. \tag{7}$$

Since [13]

$$\nu = \langle \omega^2 \rangle^{1/2} \eta^2,$$

$$\eta \propto R_\lambda^{-3/2} L, \tag{8}$$

Eq. (6) yields

$$D \propto L \sqrt{R_v / R_\lambda^3}. \tag{9}$$

Reference [9] reports that the characteristic Reynolds number R_v of the most intense vortex tubes in homogeneous isotropic turbulence grows approximately like

$$R_v \propto R_\lambda^{1/2}. \tag{10}$$

Inserting in Eq. (9), we obtain Eq. (1).

We have argued that vortex tubes sustain vortex spirals only at distances significantly smaller than D. Although passive scalar fields evolve in a different way than vorticity fields, this argument may explain also why a spiral model for passive scalar fields [3] fails when tested experimentally [11].

Finally, our estimate of the characteristic spiral diameter D does not apply to two-dimensional turbulence. Indeed, spiral models for two-dimensional turbulence [6] (see also references therein) receive some support from numerical studies.

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[1] T. S. Lundgren, Phys. Fluids **25**, 2193 (1982).
[2] H. K. Moffatt, in *Turbulence and Chaotic Phenomena in Fluids*, edited by T. Tatsumi (Elsevier, Amsterdam, 1984).
[3] J. C. Vassilicos and J. C. R. Hunt, Proc. R. Soc. London, Ser. A **435**, 505 (1991).
[4] D. I. Pullin and P. G. Saffman, Phys. Fluids **5**, 126 (1993).
[5] D. I. Pullin, J. D. Buntine, and P. G. Saffman, Phys. Fluids **6**, 3010 (1994).
[6] A. P. Bassom and A. D. Gilbert, J. Fluid Mech. **371**, 109 (1998).
[7] A. Vincent and M. Meneguzzi, J. Fluid Mech. **225**, 1 (1991).
[8] A. Vincent and M. Meneguzzi, J. Fluid Mech. **258**, 245 (1994).
[9] J. Jiménez, A. A. Wray, P. G. Saffman, and R. S. Rogallo, J. Fluid Mech. **255**, 65 (1993).
[10] T. S. Lundgren and N. N. Mansour, J. Fluid Mech. **307**, 43 (1996).
[11] R. M. Everson and K. R. Sreenivasan, Proc. R. Soc. London, Ser. A **437**, 391 (1992).
[12] G. K. Batchelor, *An Introduction to Fluid Mechanics* (Cambridge University Press, Cambridge, UK, 1967).
[13] H. Tennekes and J. L. Lumley, *A First Course in Turbulence* (MIT, Cambridge, MA, 1990).
[14] N. N. Mansour (private communication).